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Faster Algorithms for Weighted Recursive State Machines

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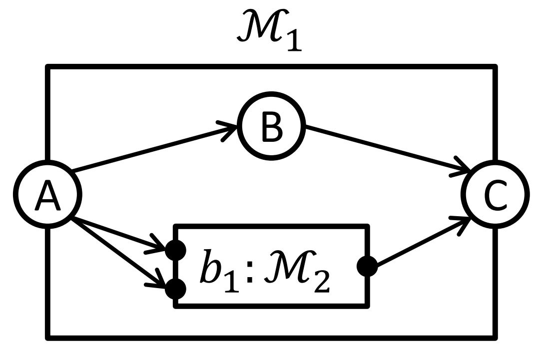
Recursive State Machines (RSMs)

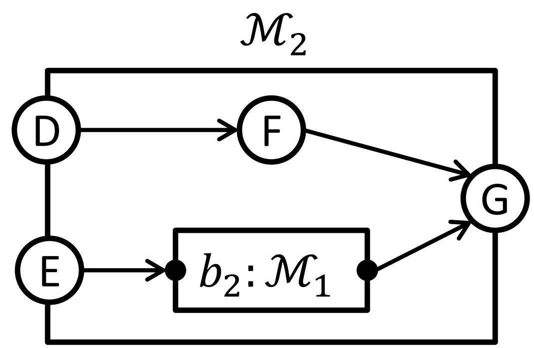
Formal model of recursive computation

Linearly equivalent to pushdown systems (PDSs)

Advantages:

- Natural modeling
- Many parameters
  - Number of modules f
  - Entry bound $\theta_e = \max_i |En_i|$
  - Exit bound $\theta_x = \max_i |Ex_i|$
  - $\theta = \max_i \min(|En_i|, |Ex_i|)$
  - …





$$\langle A, \varepsilon\rangle \Rightarrow \langle E, b_1\rangle \Rightarrow \langle A, b_2 b_1\rangle$$
$$\Rightarrow \langle B, b_2 b_1\rangle \Rightarrow \langle\langle b_2, C\rangle, b_1\rangle$$
$$\Rightarrow \langle\langle b_1, G\rangle, \varepsilon\rangle$$

RSMs over Semirings

Label RSM transitions with weights from idempotent semiring $\langle W, \oplus, \otimes, 0, 1 \rangle$

weight of computation: $\otimes$ weight of computation set: $\oplus$

	$\boldsymbol{W}$	$\oplus$	$\otimes$	**0**	**1**
Reachability	$\mathbb{B}$	$\vee$	$\wedge$	$\bot$	$\top$
Shortest path	$\mathbb{R}^+ \cup \{\infty\}$	min	$+$	∞	0
Most probable path	$[0,1]$	max	$\cdot$	0	1
IFDS	$2^D \xrightarrow{d} 2^D$	$\sqcap$	$\circ$	$\lambda x. \top$	$\lambda x. x$

Canonical partial order

$$a \leq b \Leftrightarrow a \oplus b = a$$

Monotonicity

$$a \leq b \Rightarrow a \otimes c \leq b \otimes c$$

Finite-height: $H \in \mathbb{N}$ longest descending chain in $\leq$

Distance Problems

Given a set of initial configurations S

- Configuration distance

$$d(S, \langle u, b_1 \cdots \mathrm{b}_n \rangle)$$

- Superconfiguration distance

$$d(S, \langle u, \mathcal{M}_1 \cdots \mathcal{M}_n \rangle)$$

- Node distance

$$d(S, u)$$

Our Solution

1. Configuration automata

   Representation structures for sets of RSM configurations [BEM'97]

   Initial automaton C, s.t. $\mathcal{L}(C) = S$

2. Dynamic programming algorithm

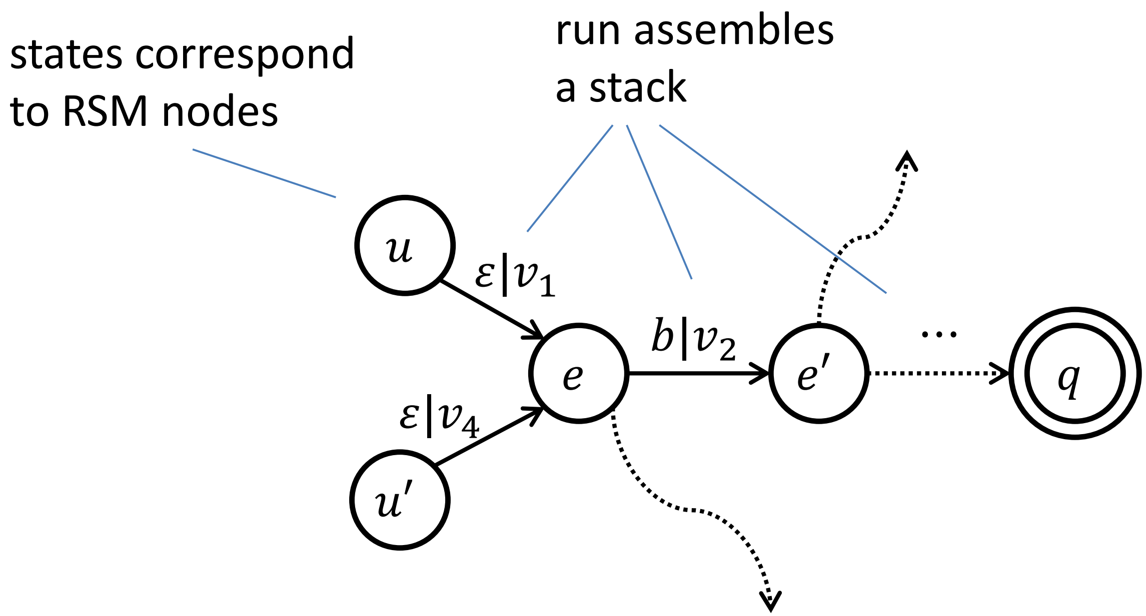
   Compute C^*, representing reachable configurations and distances

   Key: Entry-to-Exit summaries [ABEGRY'05]

3. Distance extraction algorithms

   Query configuration/superconfiguration/node distances from C^*

Configuration Automata

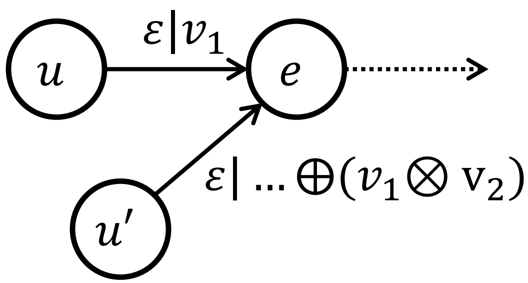


$\langle u, b \cdots \rangle$ is an accepted configuration

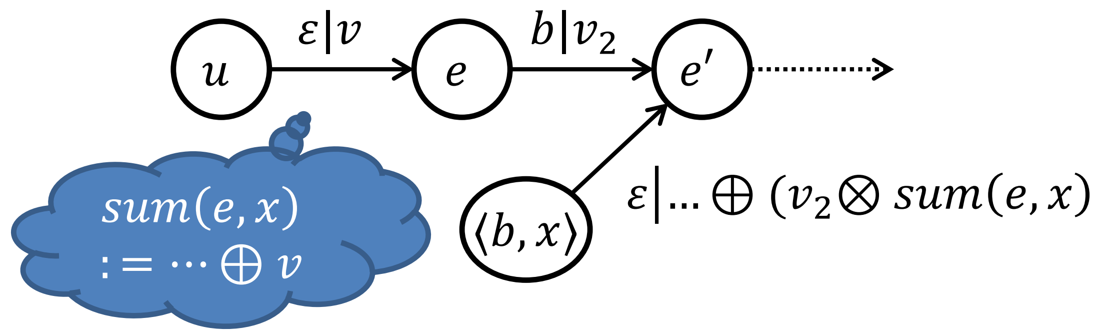
Weight of configuration is $\bigoplus$ over all accepting runs

Relaxation Steps

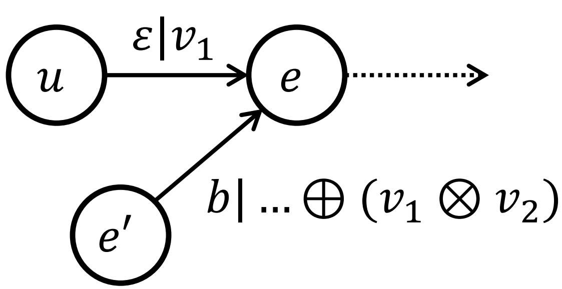
Internal transition: $u \xrightarrow{v_2} u'$



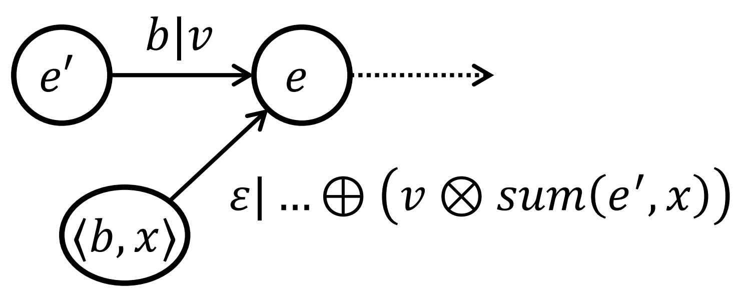
Exit transition: $u \to x$



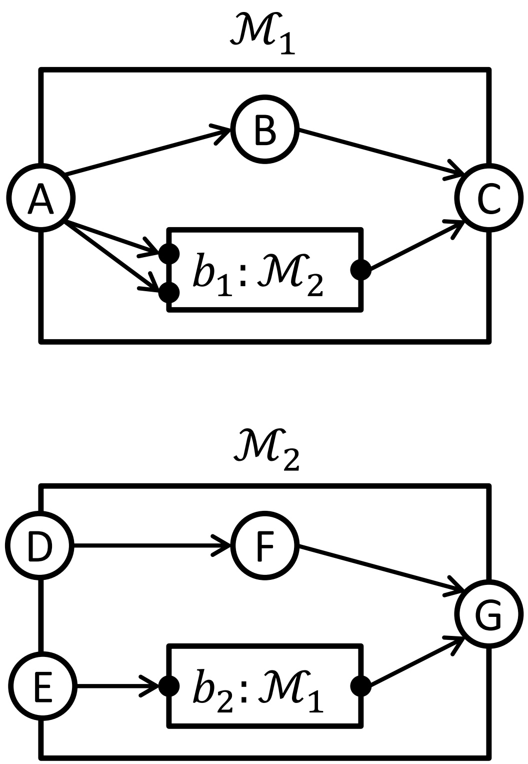
Call transition: $u \xrightarrow{v_2} \langle b, e' \rangle$

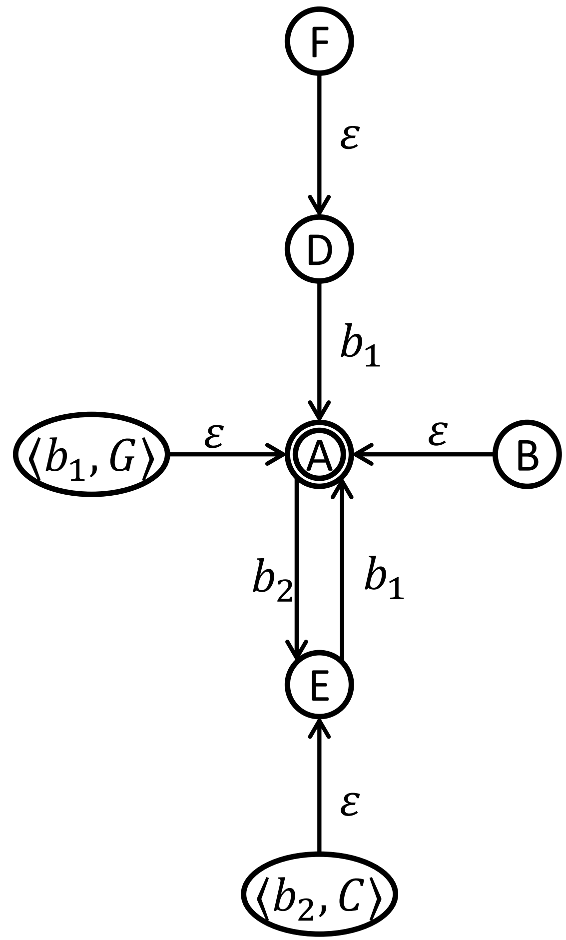


Using summary: $sum(e', x)$



Reachability Example





Summaries:

$A \rightsquigarrow C$

$E \rightsquigarrow G$

$D \rightsquigarrow G$

Correctness and Complexity

For every configuration c: $d(\mathcal{L}(C), c) = C^*(c)$

C^* is constructed in time

$$\mathcal{O}\big(H \cdot (|\mathcal{R}| \cdot \theta_e + \theta_e \cdot \theta_x \cdot |Call|)\big)$$

Compared to PDS algorithm

$$\mathcal{O}(H \cdot |\mathcal{R}| \cdot \theta_e \cdot \theta_x \cdot f)$$

Factor ${}^{|\mathcal{R}| \cdot f} \Big/ {}_{\frac{|\mathcal{R}|}{\theta_x} + |Call|}$ improvement

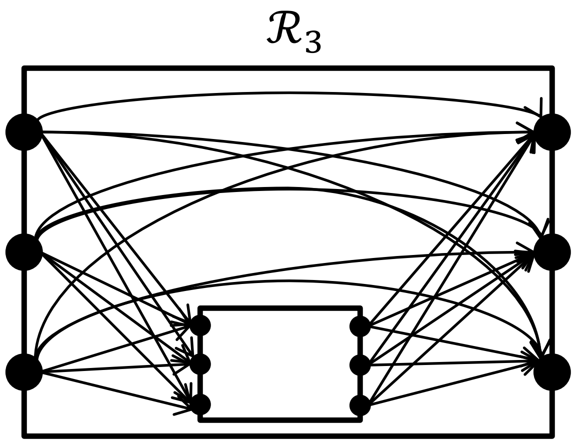
Empirical Evaluation

RSM-based algorithm vs. PDS-based algorithm

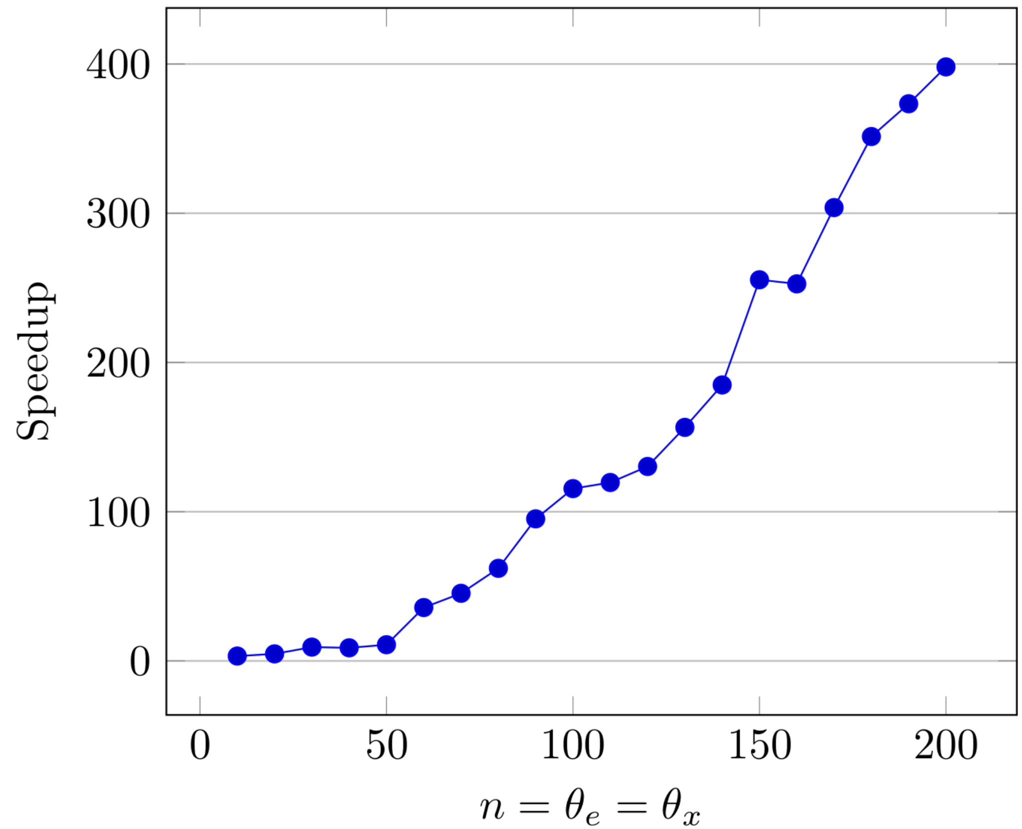
Our tool jMoped

1. Scaling on artificial examples
2. Interprocedural program analysis problems

A Family of Dense RSMs

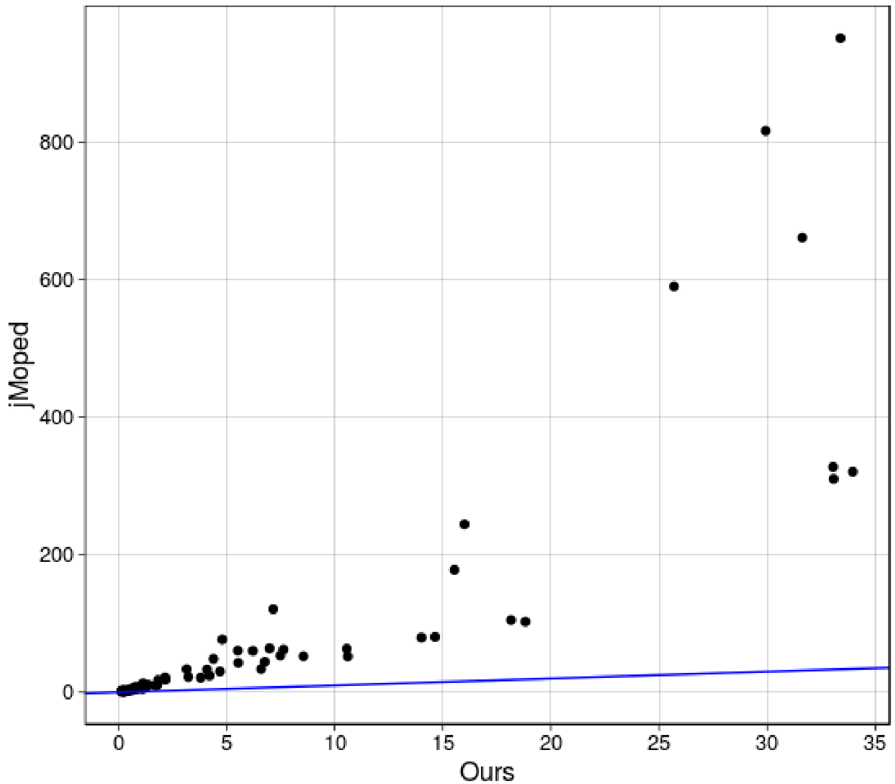


$$|\mathcal{R}| \cdot f \Big/ \frac{|\mathcal{R}|}{\theta_x} + |Call| = \frac{n^2 \cdot 1}{\frac{n^2}{n} + n} = \boldsymbol{n}$$

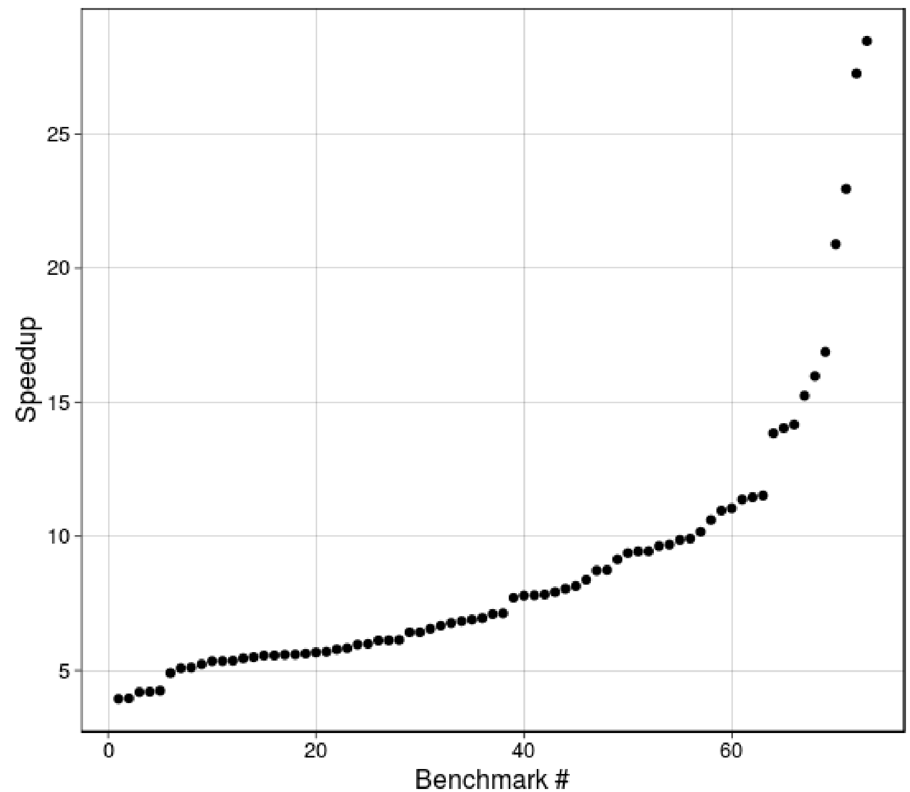


Boolean Programs from SLAM/SDV

Absolute runtime (seconds)



Relative speedup



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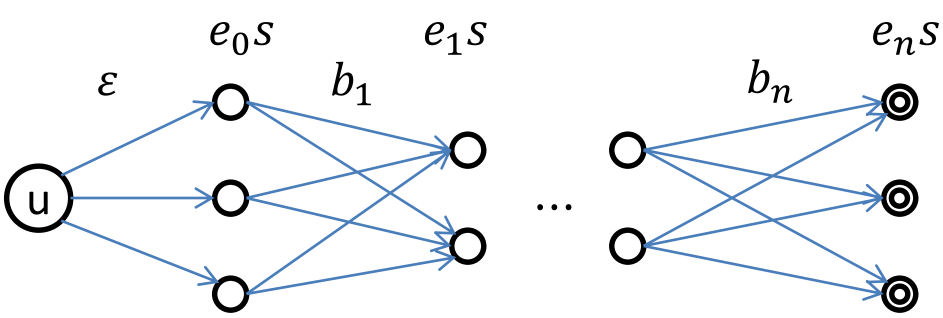
Distance Extraction

- Configuration distance for $\langle u, b_1 \cdots b_n \rangle$



Dynamic programming: $\mathcal{O}(n \cdot \theta_e^2)$

- Superconfiguration distance for $\langle u, \mathcal{M}_1 \cdots \mathcal{M}_n \rangle$

Replace $e \xrightarrow{b} e'$ with $e \xrightarrow{\mathcal{M}} e'$ where $\mathcal{M}$ is module of e'

- Node distance

Traditional single-source distance problem

Distances over Small Semirings

W^{θ_e} vector

C^* transitions $u \xrightarrow{\varepsilon} e$

$W^{\theta_e \times \theta_e}$ matrix

C^* transitions labeled b_i

$$1_{\theta_e} \cdot A_u \cdot A_{b_1} \cdots A_{b_n} \cdot 1_{\theta_e}^T$$

- Constant size semiring (Mailman's speedup)

$$\mathcal{O}\left(n \cdot \frac{\theta_e^2}{\log \theta_e}\right)$$

- Size $|W|$ semiring (William's speedup)

for $\varepsilon > 0$, $\mathcal{O}\left(n \cdot \frac{\theta_e^2}{\varepsilon^2 \log^2 \theta_e}\right)$ (some preprocessing)

- Binary semiring (Four-Russians speedup)

$$\mathcal{O}\left(|\mathcal{R}| \cdot \theta_e \cdot \frac{n}{\log n}\right)$$

Summary

- Faster interprocedural analysis (RSM > PDS)

- Configuration automata
  → Saturation algorithm (summaries)
  → Distance extraction

- More in the paper
  - Further distance extraction speedups
  - Implications for context-bounded analysis (concurrency)

Algorithm 1: `ConfDist`

Input: RSM $\mathcal{R}$ and $\mathcal{R}$-automaton $\mathcal{C}$ with $\ell(t) = \overline{1}$ for all transitions t in $\mathcal{C}$
Output: $\mathcal{R}$-automaton $\mathcal{C}_{post^*}$ with $\mathcal{C}_{post^*}(c) = d(\mathcal{L}(\mathcal{C}), c)$ for all configurations c

preprocess $\mathcal{C}$ as described in the main text
`// Initialization of worklist and summary function`
$\mathsf{WL} := \{t = q \xrightarrow{\varepsilon} q' \mid q \in I \text{ and } \ell(t) = \overline{1}\}$
$\mathsf{sum}(\langle e, m_e\rangle, x) := \overline{0}$ for all states $\langle e, m_e\rangle$ and $x \in Ex$
`// Main loop`
while $\mathsf{WL} \neq \emptyset$ **do**
    extract $t_{\mathcal{C}}$ from WL
    if $t_{\mathcal{C}} = \langle u, m_u\rangle \xrightarrow{\varepsilon} \langle e, m_e\rangle$ **then**
        let $\mathcal{M}_i$ be the module of node u
        `// Internal transitions from` u
        foreach $t_{\mathcal{R}} = \langle u, u'\rangle \in \delta_i$ *where* $u' \in In_i$ **do**
            `Relax`$(\langle u', \widehat{m}\rangle \xrightarrow{\varepsilon} \langle e, m_e\rangle, \ell(t_{\mathcal{C}}) \otimes w_i(t_{\mathcal{R}}))$
        `// Call transitions from` u
        foreach $t_{\mathcal{R}} = \langle u, \langle b, e'\rangle\rangle \in \delta_i$ **do**
            `Relax`$(\langle e', \widehat{m}\rangle \xrightarrow{b} \langle e, m_e\rangle, \ell(t_{\mathcal{C}}) \otimes w_i(t_{\mathcal{R}}))$
            add $\langle e', \widehat{m}\rangle \xrightarrow{\varepsilon} \langle e', \widehat{m}\rangle$ to WL, if it was never added before
        `// Exit transitions from` u
        foreach $t_{\mathcal{R}} = \langle u, x\rangle \in \delta_i$ *where* $x \in Ex_i$ **do**
            if $\mathsf{sum}(\langle e, m_e\rangle, x) \not\sqsubseteq \ell(t_{\mathcal{C}})$ **then**
                $\mathsf{sum}(\langle e, m_e\rangle, x) := \mathsf{sum}(\langle e, m_e\rangle, x) \oplus \ell(t_{\mathcal{C}})$
                foreach $\langle e, m_e\rangle \xrightarrow{b/v} \langle e', m_{e'}\rangle$ **do**
                    `Relax`$(\langle\langle b, x\rangle, \widehat{m}\rangle \xrightarrow{\varepsilon} \langle e', m_{e'}\rangle, v \otimes \mathsf{sum}(\langle e, m_e\rangle, x))$
    else if $t_{\mathcal{C}} = \langle e, m_e\rangle \xrightarrow{b} \langle e', m_{e'}\rangle$ **then**
        let $\mathcal{M}_i$ be the module of node e
        `// Using entry-to-exit summaries`
        foreach $x \in Ex_i$ **do**
            `Relax`$(\langle\langle b, x\rangle, \widehat{m}\rangle \xrightarrow{\varepsilon} \langle e', m_{e'}\rangle, \ell(t_{\mathcal{C}}) \otimes \mathsf{sum}(\langle e, \widehat{m}\rangle, x)$

Procedure `Relax`(t, v)
    if $\ell(t) \neq \ell(t) \oplus v$ **then**
        $\ell(t) := \ell(t) \oplus v$
        add t to WL