Tree Interpolation in Vampire

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Bernhard Kragl (TU Vienna)

Interpolation

Craig/Binary Interpolant

$$A \quad \wedge \quad B \quad \rightarrow \quad \bot$$

$$I$$

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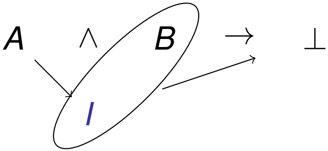

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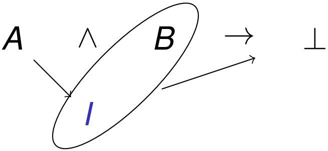

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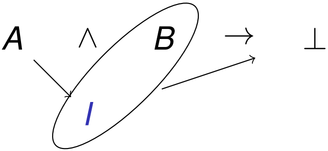

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Sequence Interpolant

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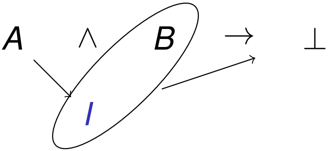

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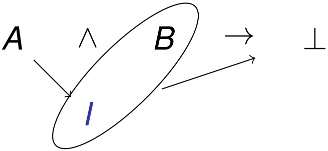

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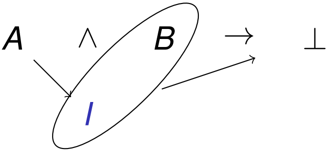

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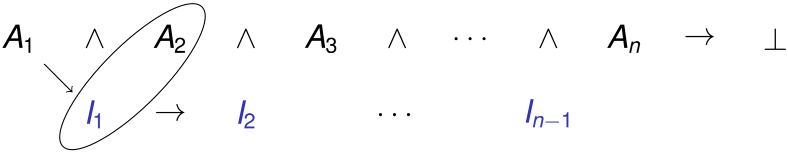

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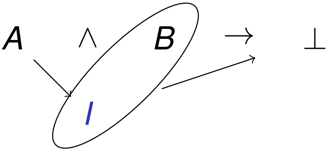

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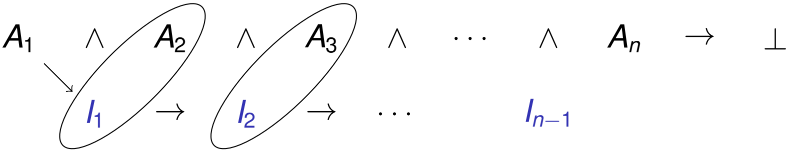

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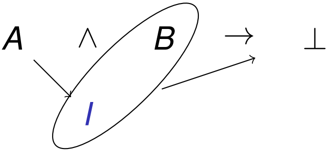

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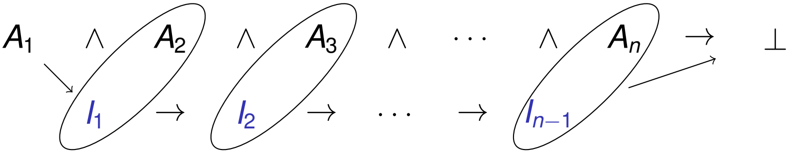

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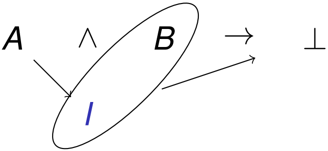

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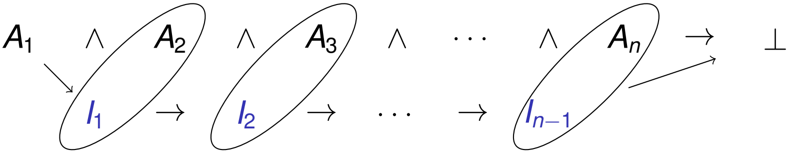

$I_1 \in \mathcal{L}(A_1) \cap \mathcal{L}(A_2, \ldots, A_n)$

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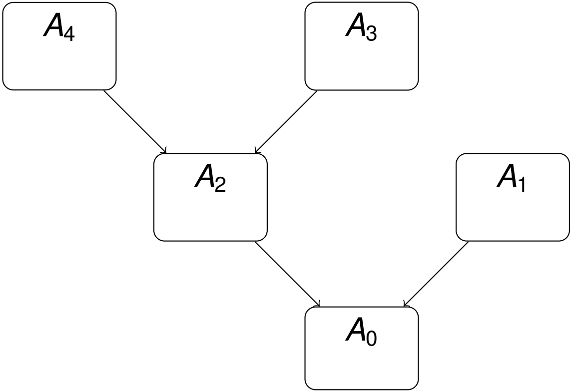
$I_{n-1} \in \mathcal{L}(A_1, \ldots, A_{n-1}) \cap \mathcal{L}(A_n)$

Tree interpolation

$$A_0 \wedge A_1 \wedge A_2 \wedge A_3 \wedge A_4 \rightarrow \bot$$



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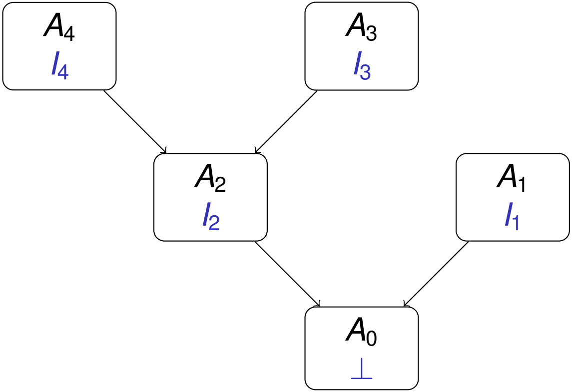

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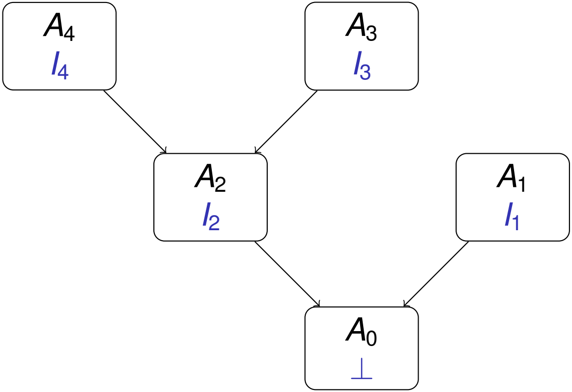

node $\wedge$ child Itps $\rightarrow$ Itp

$$A_4 \rightarrow I_4$$
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$$A_2 \wedge I_4 \wedge I_3 \rightarrow I_2$$
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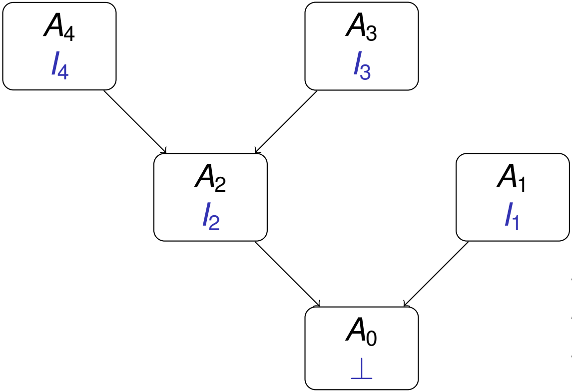

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$$A_2 \wedge I_4 \wedge I_3 \rightarrow I_2$$
$$A_0 \wedge I_2 \wedge I_1 \rightarrow \bot$$

Language restrictions

$$I_4 \in \mathcal{L}(A_4) \cap \mathcal{L}(A_3, A_2, A_1, A_0)$$
$$I_3 \in \mathcal{L}(A_3) \cap \mathcal{L}(A_4, A_2, A_1, A_0)$$
$$I_2 \in \mathcal{L}(A_4, A_3, A_2) \cap \mathcal{L}(A_1, A_0)$$
$$I_1 \in \mathcal{L}(A_1) \cap \mathcal{L}(A_3, A_2, A_1, A_0)$$

Related Work

Solving recursion-free Horn clauses
[Gupta, Popeea, Rybalchenko POPL'11]

Interpolants for procedure summarization
[McMillan, Rybalchenko MSR-TR'13]

Generalized property directed reachability
[Hoder, Bjørner SAT'12]

Interpolation and Horn Clauses
[Hojjat, Rümmer, Kuncak CAV'13]

Nested Interpolants
[Heizmann, Hoenicke, Podelski POPL'10]

and many more ...

Important questions

- Do interpolants always exist?
  Yes, in first-order logic (also with respect to a theory)
- Is a logic closed under interpolation? (e.g. quantifier free fragments)
  Not necessarily, consider $a = 2b + 1 \wedge a = 2c$ over $\mathbb{Z}$

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- How to interpolate efficiently?
- How to obtain "good" interpolants?

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- Refutations (should) capture the cause of unsatisfiability
- Extensive literature on interpolant extraction for various theories
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interpolant: $a = c$

Vampire

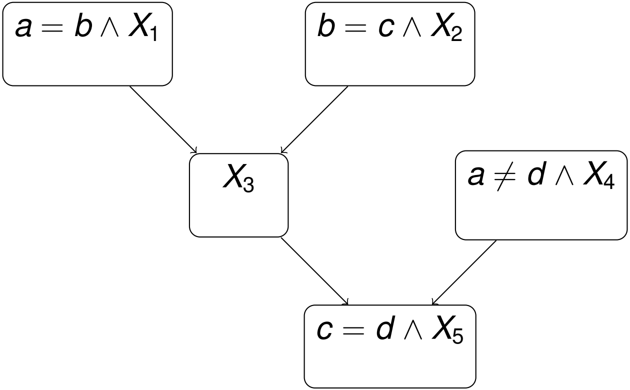
- Vampire is one of the best first-order theorem provers

- Recent developments/extensions:
  - Invariant generation [FASE'09, MICAI'11]
  - Interpolation and Symbol Elimination [CADE'09, IJCAR'10]
  - Interpolant minimization &
    Theory independent proof localization [POPL'12]
  - Incremental tree interpolation [today]

Incremental tree interpolation

- Visit tree nodes in *topological order*
- Per node: *partition* the tree and compute *binary interpolant*
- Crucial: reuse previously computed interpolants

$\mathcal{L}(X_1, X_2, X_3, X_4, X_5) \cap \{a, b, c, d\} = \emptyset$



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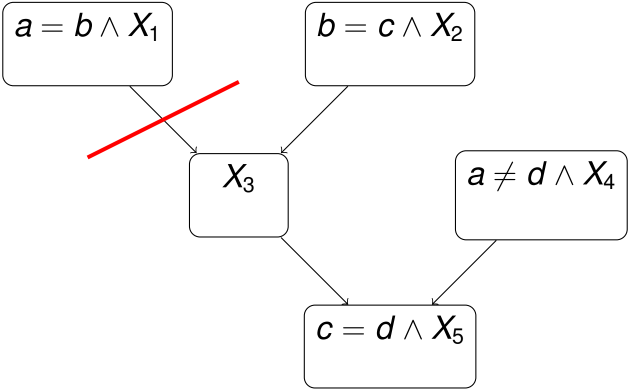

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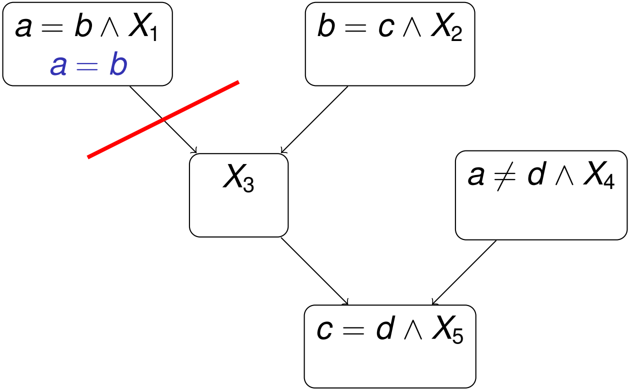

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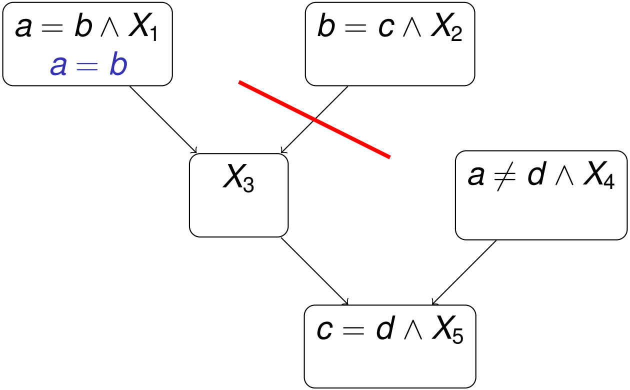

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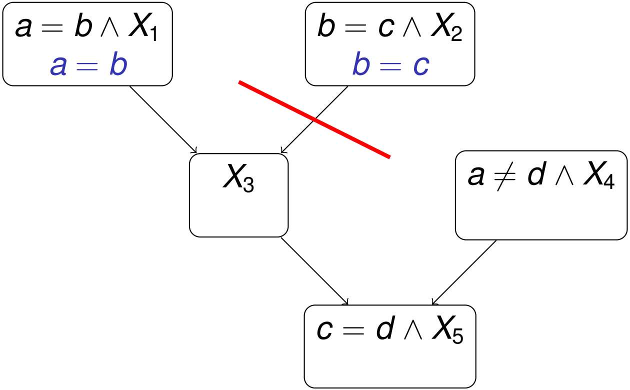

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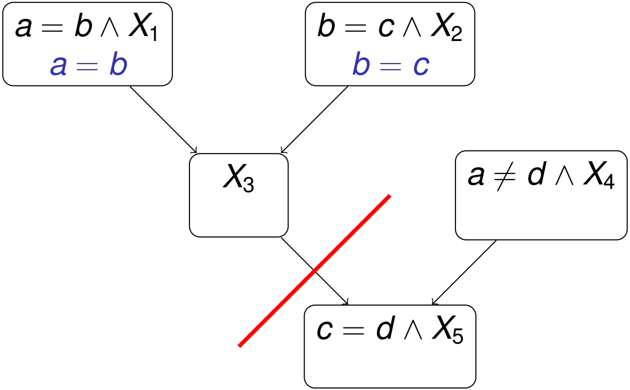

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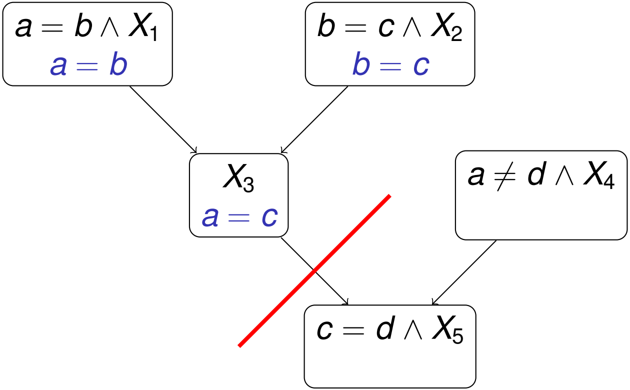

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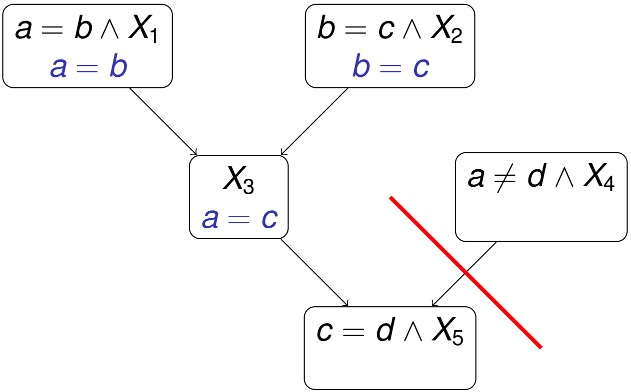

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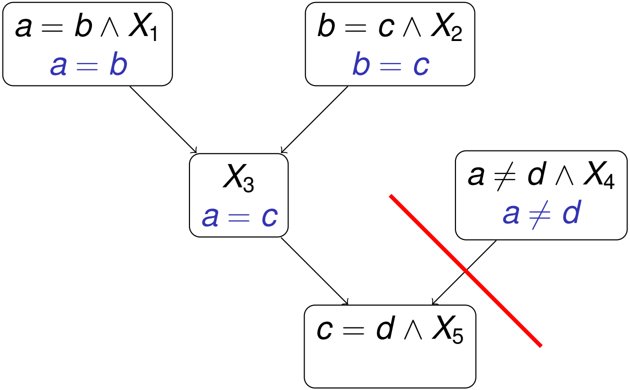

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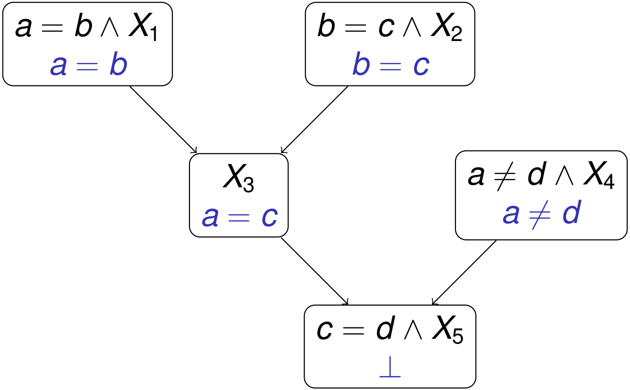

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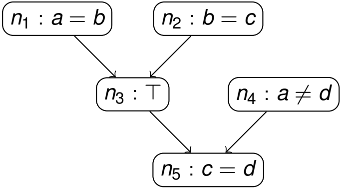

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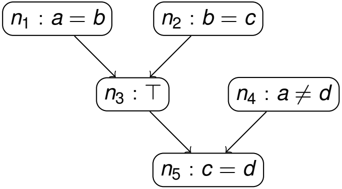

Tree interpolation problem in SMT-LIB 1.2 syntax using iZ3 convention

```
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:assumption (implies (and
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:assumption (implies (and
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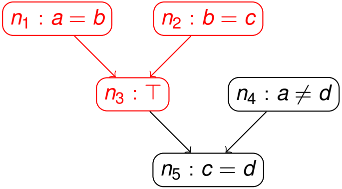

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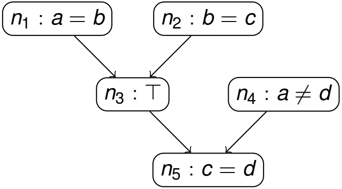


```
> vampire --show_interpolant tree x.smt
Parsing SMTLIB file: x.smt
Parsing terminated.
Building Tree.
Building Tree terminated.

n1: (= a b)
n2: (= b c)
n3: (= a c)
n4: (not (= a d))
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Evaluation

Quantifier-free benchmarks

175 QF_AUFLIA problems from model checking Windows device drivers (90 nodes on average)

Quantified benchmarks

4 small AUFLIA problems

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Vampire/Z3	113	
iZ3	175	

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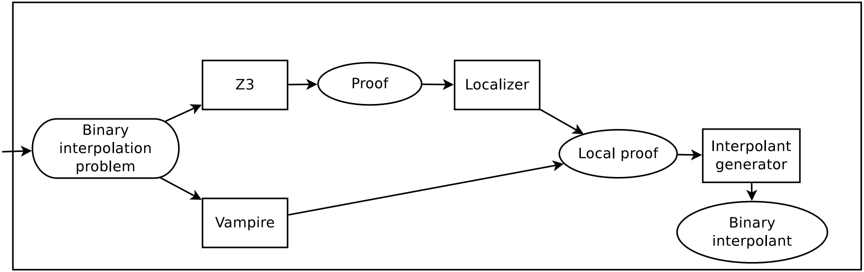

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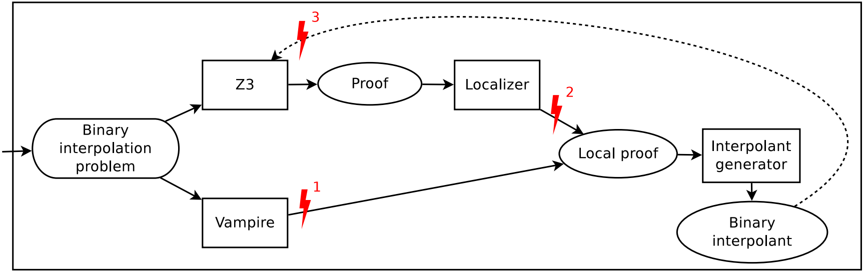

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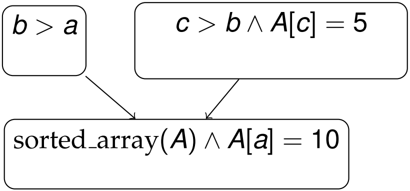

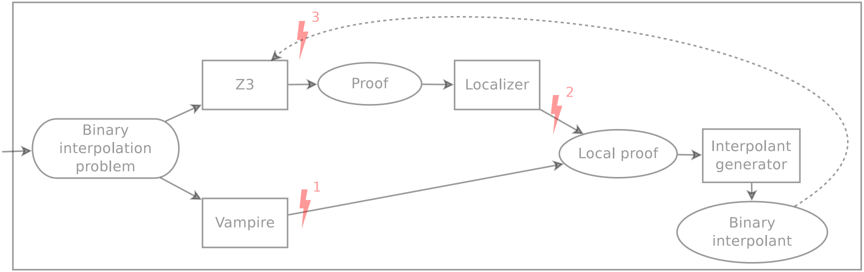

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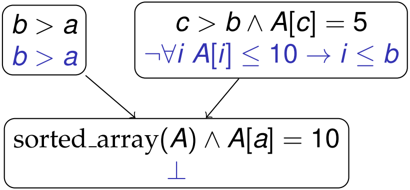

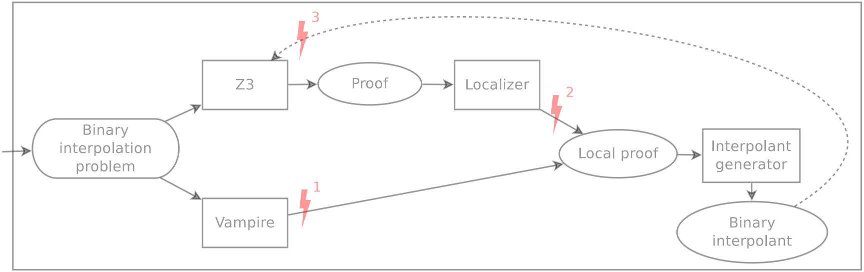

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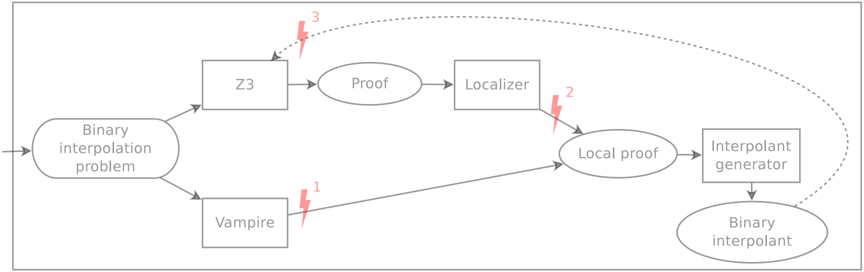
Vampire/Vampire	4
iZ3	1



Conclusion

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- Visit, try, utilize!
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Thank you!